

Transmission of Analog Signal - II

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Outline



- Explain the basic concept of *angle modulation*.
- Distinguish between FM and PM.
- Explain the basic concepts of *digital data to analog signal* conversion
- Explain different aspects of ASK, FSK, PSK and QAM conversion techniques.
- Explain bandwidth and power requirement.

Angle Modulation

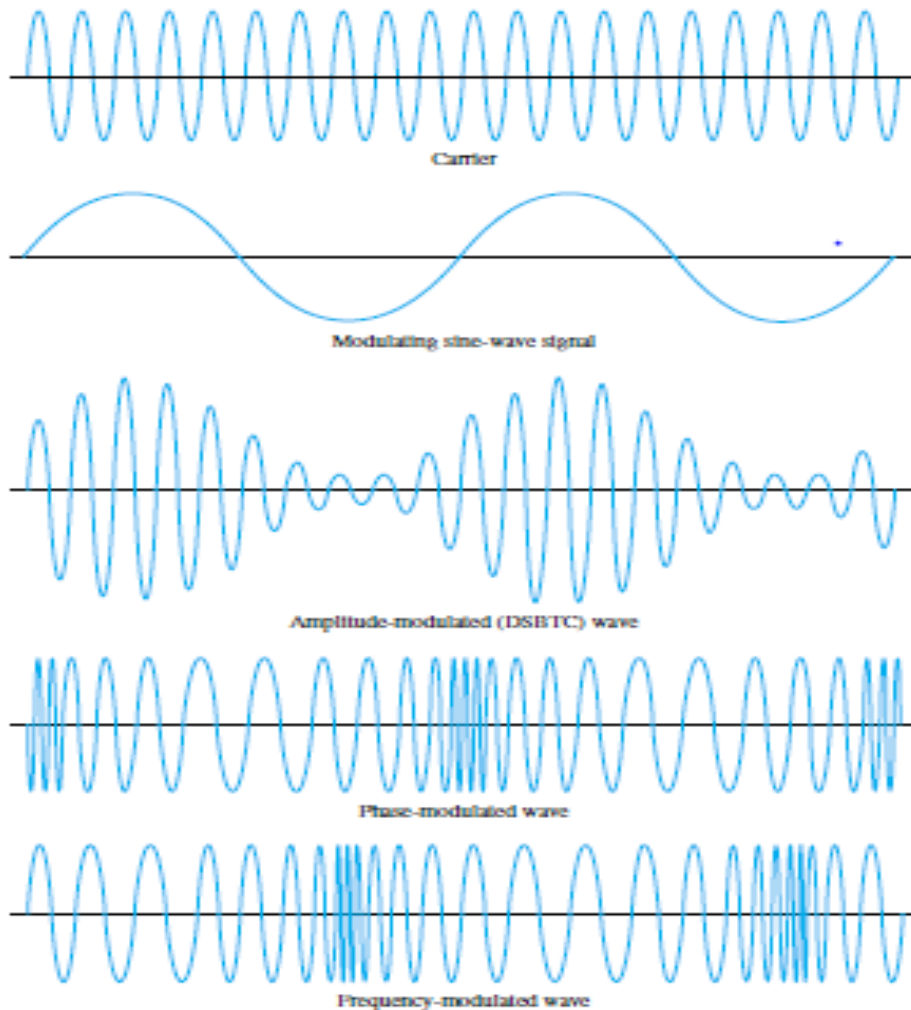


Figure 5.24 Amplitude, Phase, and Frequency Modulation of a Sine-Wave Carrier by a Sine-Wave Signal

Angle Modulation:

1. FM (Frequency Modulation)
2. PM (Phase Modulation)

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

Frequency Modulation

- The **modulating signal** $e_m(t)$ is used to vary the carrier frequency.
- The **change in frequency** is proportional to the modulating voltage $k.e_m(t)$, k is the constant known as **frequency deviation constant**, expressed in Hz/V.
- The **instantaneous frequency** of the modulated signal is $f_i(t) = f_c + k.e_m(t)$, where f_c is the carrier frequency.

Sinusoidal FM



- For sinusoidal modulation

$$e_m(t) = E_m \cos 2\pi f_m t$$

Instantaneous frequency:

$$f_i(t) = f_c + k \cdot e_m(t) = f_c + k \cdot E_m \cos 2\pi f_m t = f_c + \Delta f \cos 2\pi f_m t$$

Modulated Signal:

$$\begin{aligned} s(t) &= E_c \cos \theta(t) \\ &= E_c \cos(2\pi f_c t + 2\pi \Delta f \int_0^t \cos 2\pi f_m t \, dt) \\ &= E_c \cos(2\pi f_c t + (\Delta f / f_m) \sin 2\pi f_m t) \end{aligned}$$

The **modulation index**, denoted by β , is given by $\beta = (\Delta f / f_m)$

$$\text{Or, } s(t) = E_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

Bandwidth

- The modulated signal will contain frequency components $f_c + f_m$, $f_c + 2f_m$, and so on

- Carson's Rule

$$B_T = 2(\beta + 1)B_m$$

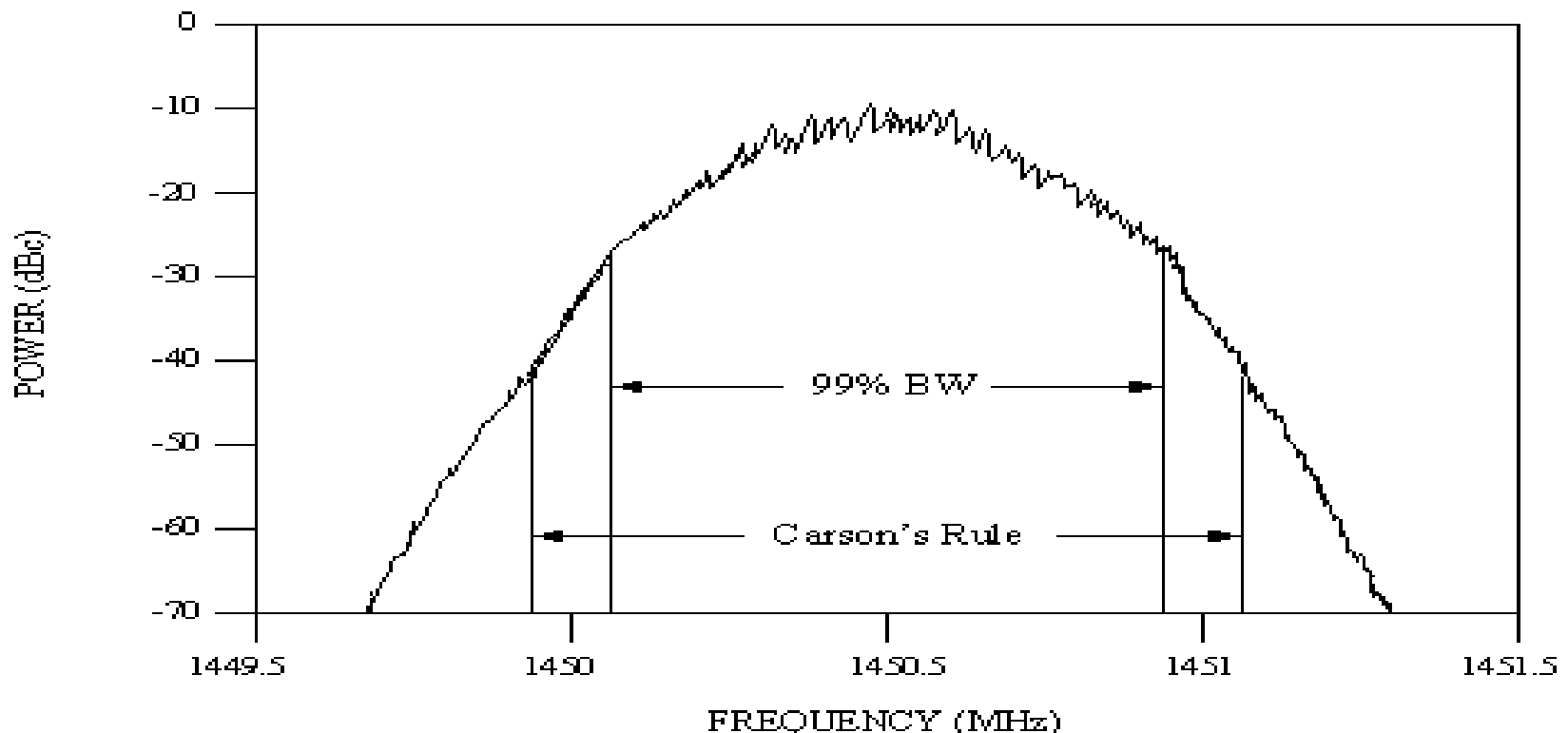
$$\text{Where, } \beta = \Delta f / B = n_f A_m / 2\pi B$$

$$B_T = 2\Delta f + 2B$$

- FM requires greater bandwidth than AM

Cont...

- Peak Deviation $= \Delta f = (1/2\pi) n_f A_m$ Hz
where A_m is the maximum value of $m(t)$



Power

- As the amplitude remains constant, **total average power** is equal to the unmodulated carrier power.
- $\text{Power} = A_c^2/2$
- Although A_m increases the bandwidth, it does not affect power.
- Transmission power for FM is less at the expense of high bandwidth.

Phase Modulation

- Representation of modulated signal
$$s(t) = A_c \cos[w_c t + \phi(t)]$$
- The angle $w_c t + \phi(t)$ goes under modulation around the angle $\theta = w_c t$
- The signal is therefore an angular-velocity modulated signal.
- When the **phase is directly proportional to the modulating signal**, i.e, $\phi(t) = n_p m(t)$, we call it phase modulation, where n_p is phase modulation index.

Relation Between FM and PM

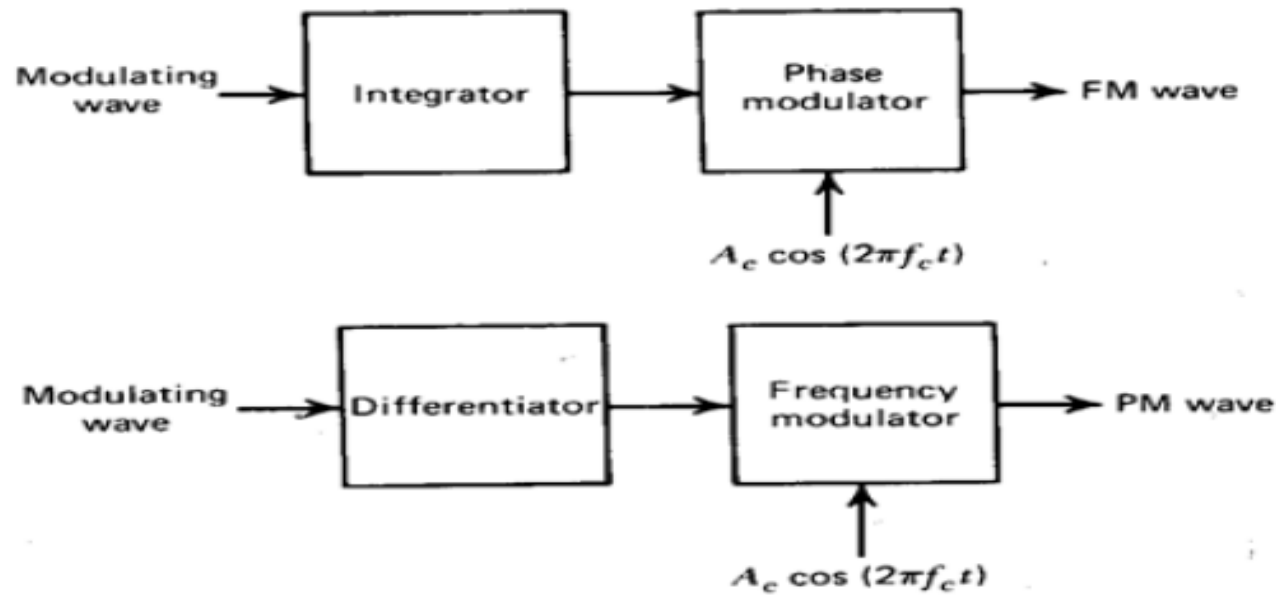


Fig: 5.2 – Scheme for generation of FM and PM Waveforms

The instantaneous frequency of the phase modulated signal

$s(t) = E_c \cos[w_c t + k' m(t)]$, Where k' is constant.

Cont...

- Let $m(t)$ be derived as an integral of the modulated signal $e_m(t)$, so that, $m(t)=k'' \int e(t)$,
- then with $k=k'k''$, we get, $s(t)=E_c \cos(\omega_c t + k \int e(t))$,
- The instantaneous angular frequency of $s(t)$ is
$$2\pi f_i(t) = d/dt(2\pi f_c t + k \int e(t))$$
or
$$f_i(t) = f_c + (1/2\pi)k e(t)$$
- The waveform is therefore modulated in frequency
- In summary, these two together are referred to as **angle modulation** and modulated signals have similar characteristics.

Angle Modulation



Angle Modulation

Frequency modulation (FM) and phase modulation (PM) are special cases of angle modulation. The modulated signal is expressed as

$$\text{Angle Modulation} \quad s(t) = A_c \cos[2\pi f_c t + \phi(t)] \quad (5.13)$$

For phase modulation, the phase is proportional to the modulating signal:

$$\text{PM} \quad \phi(t) = n_p m(t) \quad (5.14)$$

where n_p is the phase modulation index.

For frequency modulation, the derivative of the phase is proportional to the modulating signal:

$$\text{FM} \quad \phi'(t) = n_f m(t) \quad (5.15)$$

where n_f is the frequency modulation index and $\phi'(t)$ is the derivative of $\phi(t)$.

EXAMPLE 5.5 Derive an expression for $s(t)$ if $\phi(t)$ is the phase-modulating signal $n_p \cos 2\pi f_m t$. Assume that $A_c = 1$. This can be seen directly to be

$$s(t) = \cos[2\pi f_c t + n_p \cos 2\pi f_m t]$$

The instantaneous phase deviation from the carrier signal is $n_p \cos 2\pi f_m t$. The phase angle of the signal varies from its unmodulated value in a simple sinusoidal fashion, with the peak phase deviation equal to n_p .

Using Bessel's trigonometric identities, we get

$$s(t) = J_0(n_p) \cos 2\pi f_c t + \sum_{n=1}^{\infty} J_n(n_p) \left[\cos \left(2\pi(f_c + n f_m)t + \frac{n\pi}{2} \right) + \cos \left(2\pi(f_c - n f_m)t + \frac{(n+2)\pi}{2} \right) \right]$$

The resulting signal has a component at the original carrier frequency plus a set of sidebands displaced from f_c by all possible multiples of f_m . For $n_p \ll 1$, the higher-order terms fall off rapidly.

EXAMPLE 5.6 Derive an expression for $s(t)$ if $\phi'(t)$ is the frequency modulating signal $-n_f \sin 2\pi f_m t$. The form of $\phi'(t)$ was chosen for convenience. We have

$$\phi(t) = - \int n_f \sin 2\pi f_m t \, dt = \frac{n_f}{2\pi f_m} \cos 2\pi f_m t$$

Thus

$$\begin{aligned} s(t) &= \cos \left[2\pi f_c t + \frac{n_f}{2\pi f_m} \cos 2\pi f_m t \right] \\ &= \cos \left[2\pi f_c t + \frac{\Delta F}{f_m} \cos 2\pi f_m t \right] \end{aligned}$$

The instantaneous frequency deviation from the carrier signal is $-n_f \sin 2\pi f_m t$. The frequency of the signal varies from its unmodulated value in a simple sinusoidal fashion, with the peak frequency deviation equal to n_f radians/second.

The equation for the FM signal has the identical form as for the PM signal, with $\Delta F/f_m$ substituted for n_p . Thus the Bessel expansion is the same.

Bandwidth

- In the most general case, infinite bandwidth is required to transmit an FM or PM signal. As a practical matter, a very good rule of thumb, known as Carson's rule is

$$B_T = 2(\beta + 1)B$$

where

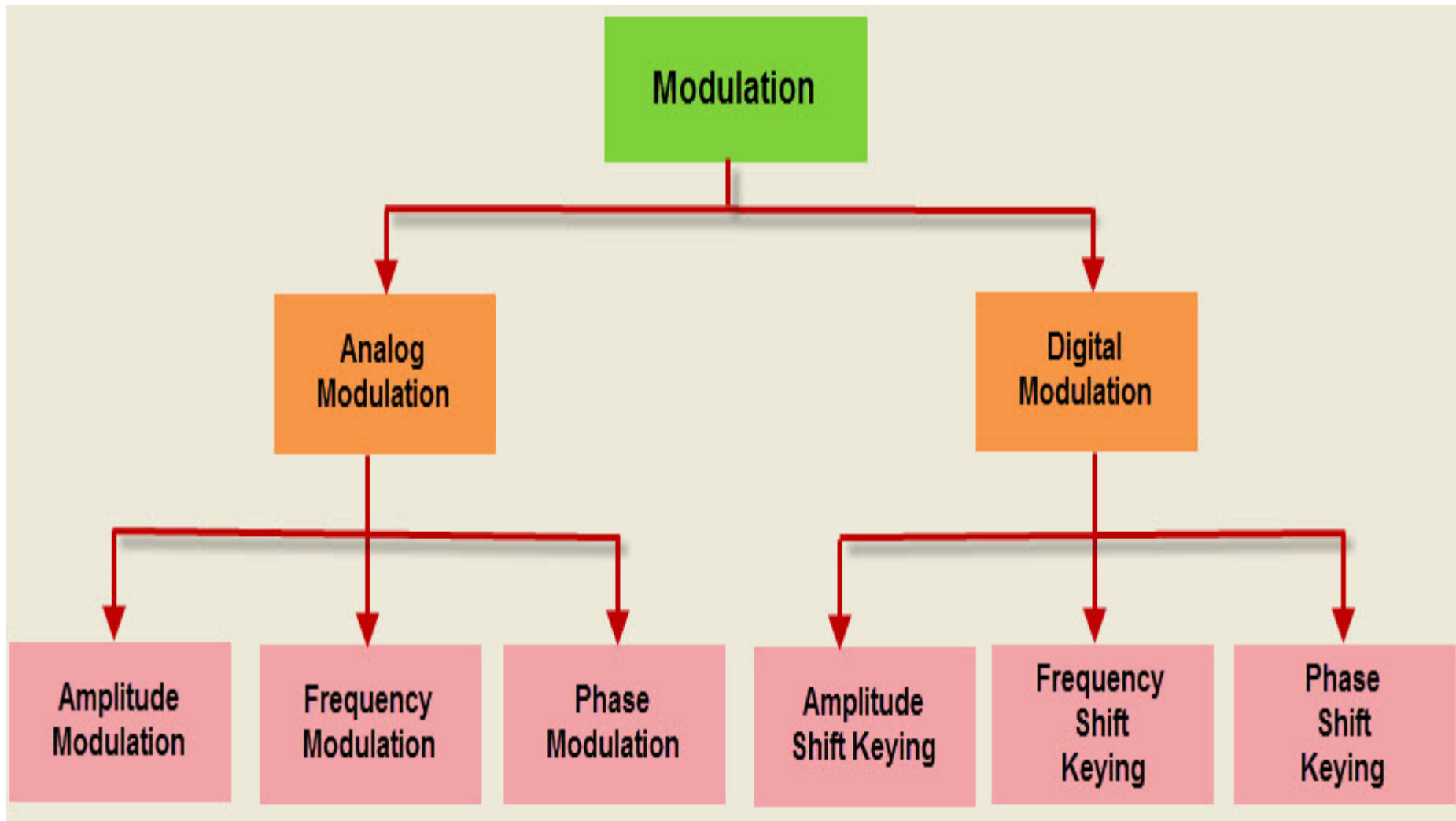
$$\beta = \begin{cases} n_p A_m & \text{for PM} \\ \frac{\Delta F}{B} = \frac{n_f A_m}{2\pi B} & \text{for FM} \end{cases}$$

We can rewrite the formula for FM as

$$B_T = 2\Delta F + 2B$$

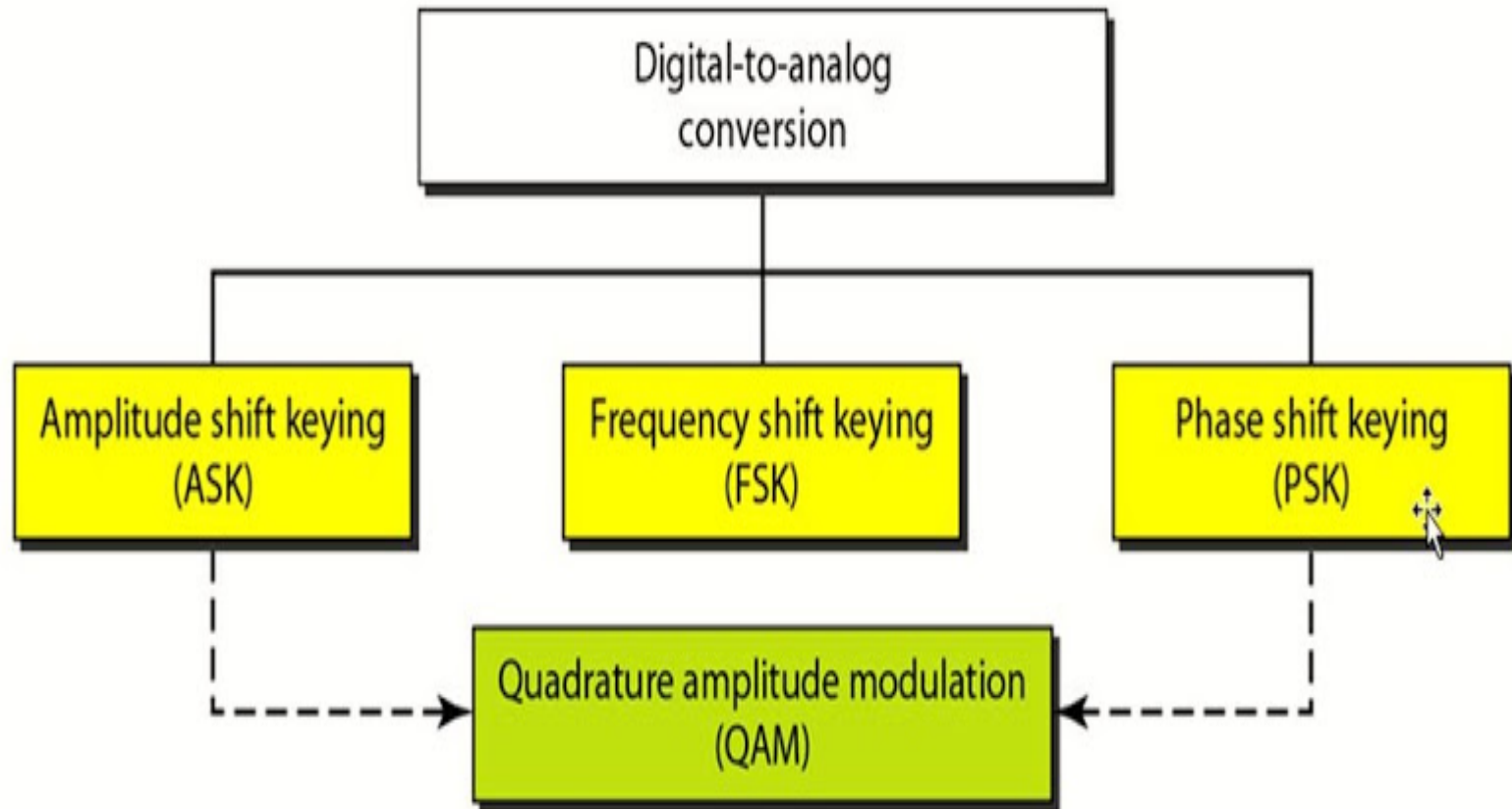
Thus both FM and PM require greater bandwidth than AM.

Modulation Technique



Digital Data-Analog Signal

- Types of digital-to-analog modulation



Amplitude Shift Keying (ASK)

- The unmodulated signal can be represented by
$$e_c(t) = E_c \cos 2\pi f_c t$$

- The modulated signal can be written as
$$s(t) = k e_m \cos 2\pi f_c t$$

$$s(t) = A_1 \cos 2\pi f_c t \quad \text{for } 1$$

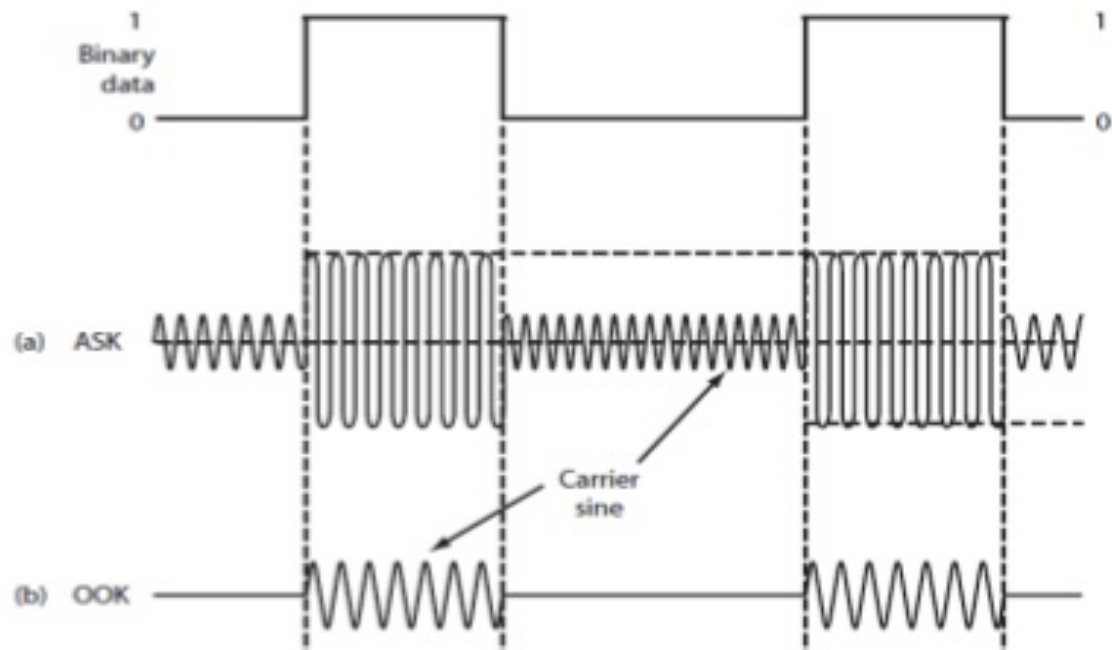
$$s(t) = A_2 \cos 2\pi f_c t \quad \text{for } 0$$

Special case: on off keying (OOK), A_2 is 0

- ASK** is susceptible to sudden gain change.
- OOK** is used to transmit digital data over optical fibers.

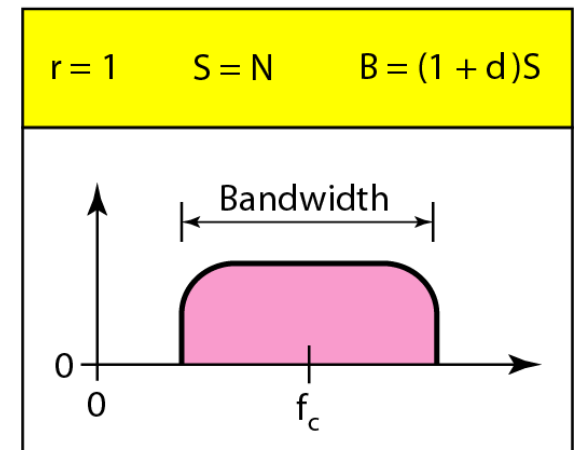
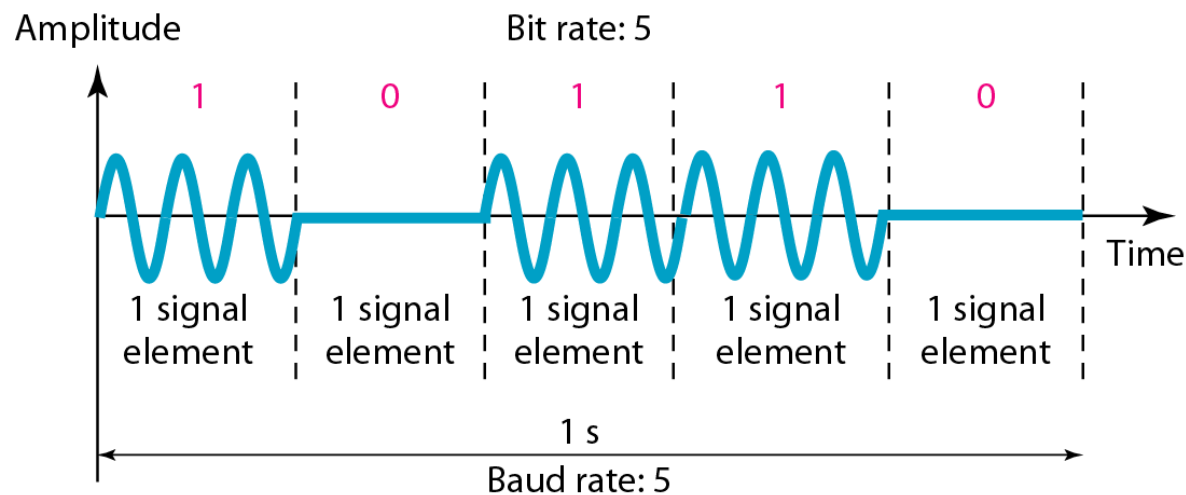
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OOK



Frequency Spectrum of ASK Signal

- If B_m is the bandwidth of the binary signal, the bandwidth of the modulated signal is $B_T = S$, where S is the baud rate.



In fact, $B = (1+d)S$ where, d is due to modulation and filtering, lies between 0 and 1.

Frequency Shifting Keying



- Frequency of the carrier is used to represent 0 or 1.

$$s(t) = A \cos 2\pi f_{c1} t \quad \text{for binary } 1$$

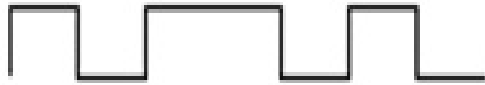
$$s(t) = A \cos 2\pi f_{c2} t \quad \text{for binary } 0$$

- It is much less susceptible to noise and gain change.

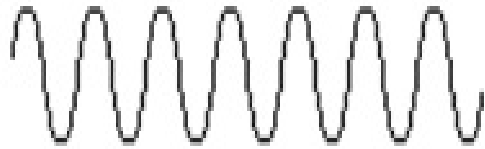
FSK

1 0 1 1 0 1 0

digital data



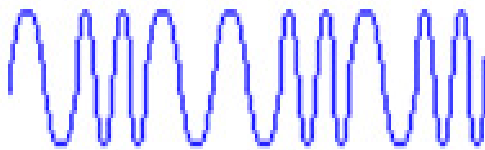
modulating signal



carrier signal n. 1

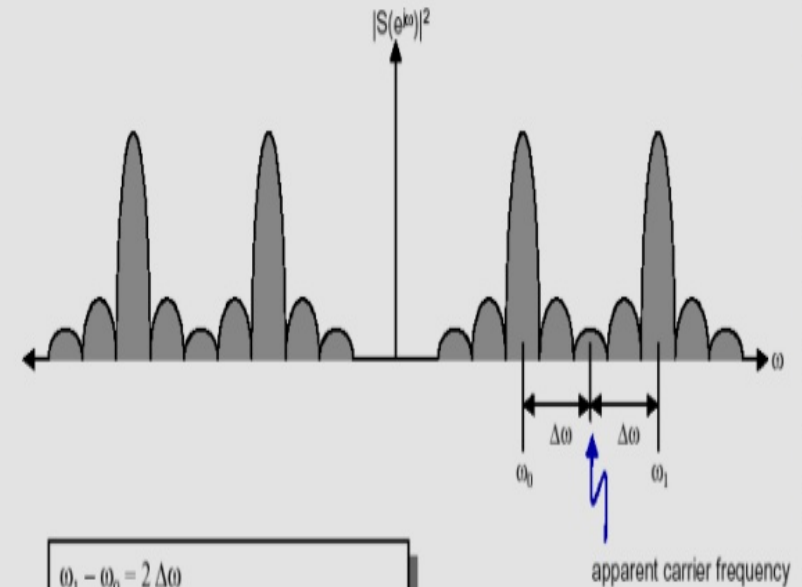


carrier signal n. 2



FSK modulated signal

FSK Spectrum

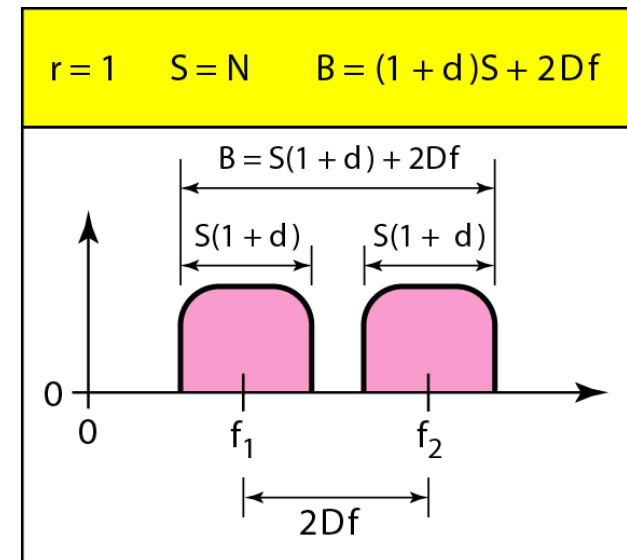
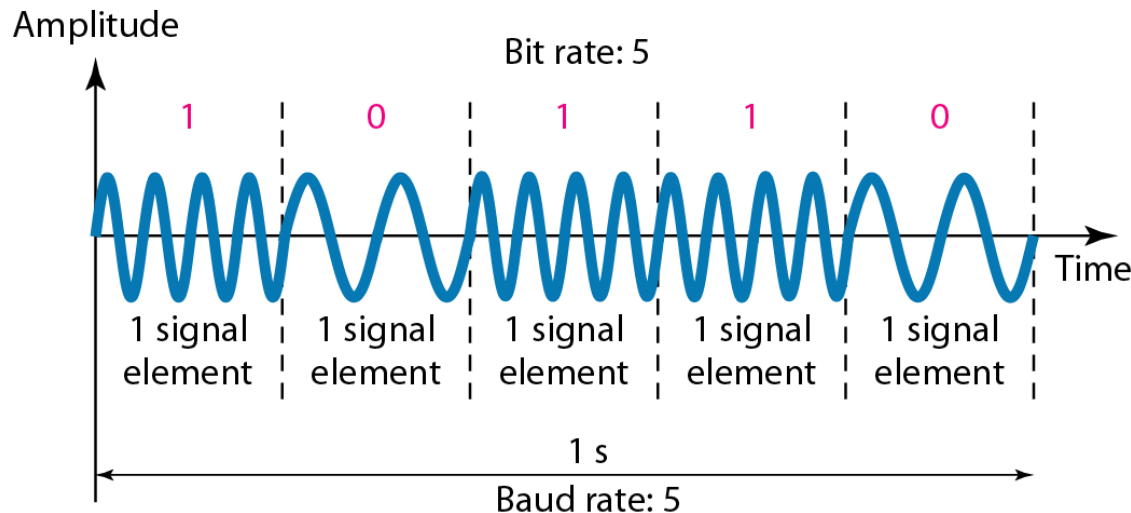


$$\omega_1 - \omega_0 = 2 \Delta\omega$$

where
 $\Delta\omega$ = frequency shift from apparent carrier

Frequency Spectrum of The FSK Signal

- FSK may be consider as combination of two ASK spectra centered around f_{c1} and f_{c2} .
- Bandwidth= $f_{c2} - f_{c1} + N_b$



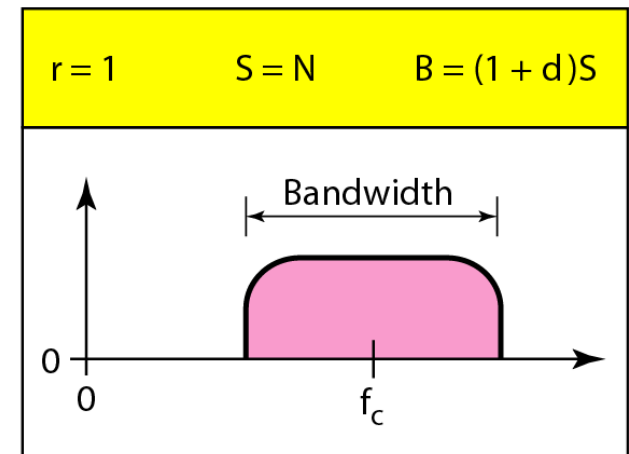
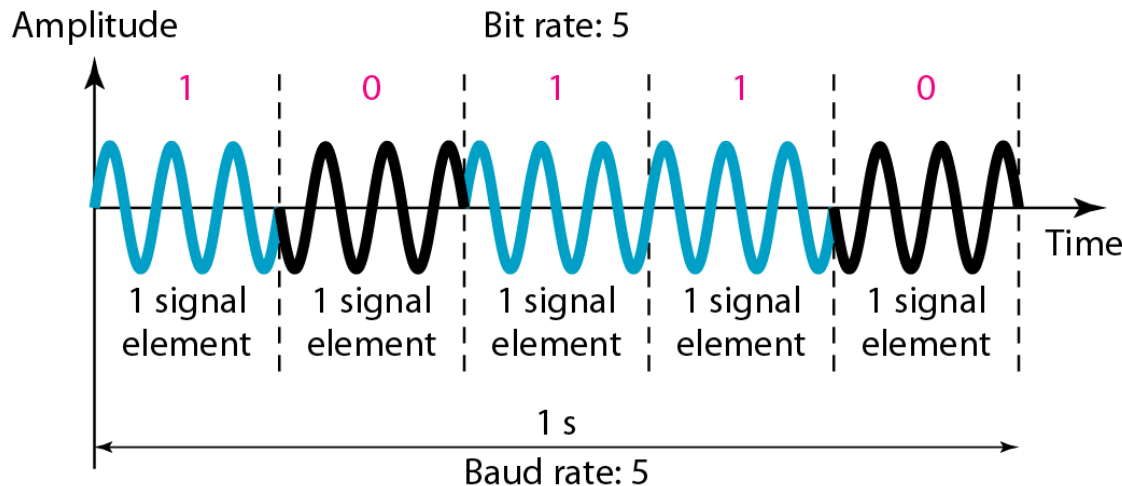
Phase Shifting Keying (PSK)

- The phase of carrier is used to represent 0 or 1

$$s(t) = A \cos(2\pi f_c t + \pi) \quad \text{for binary } 1$$

$$s(t) = A \cos 2\pi f_c t \quad \text{for binary } 0$$

• 2-PSK



Example

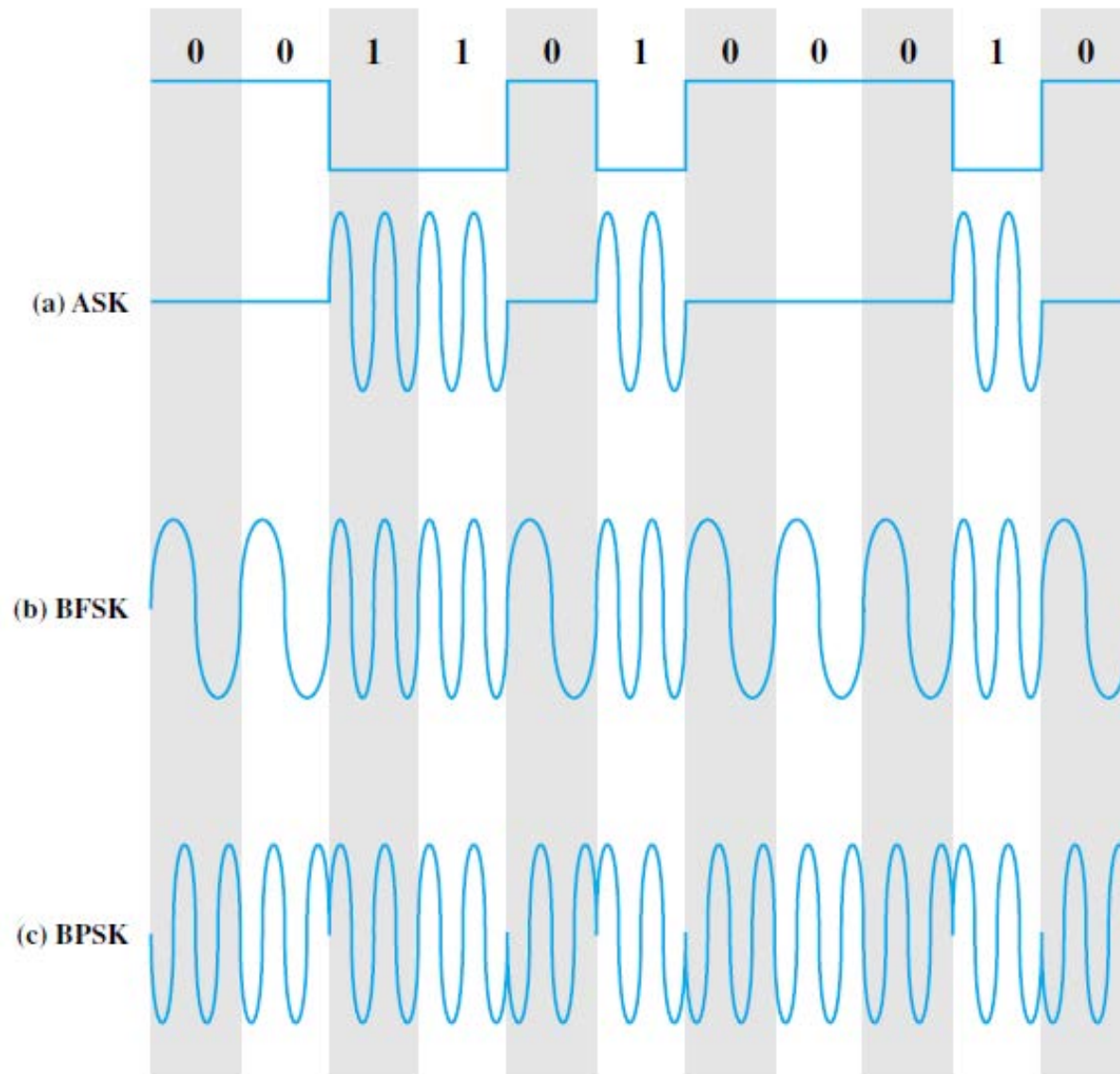
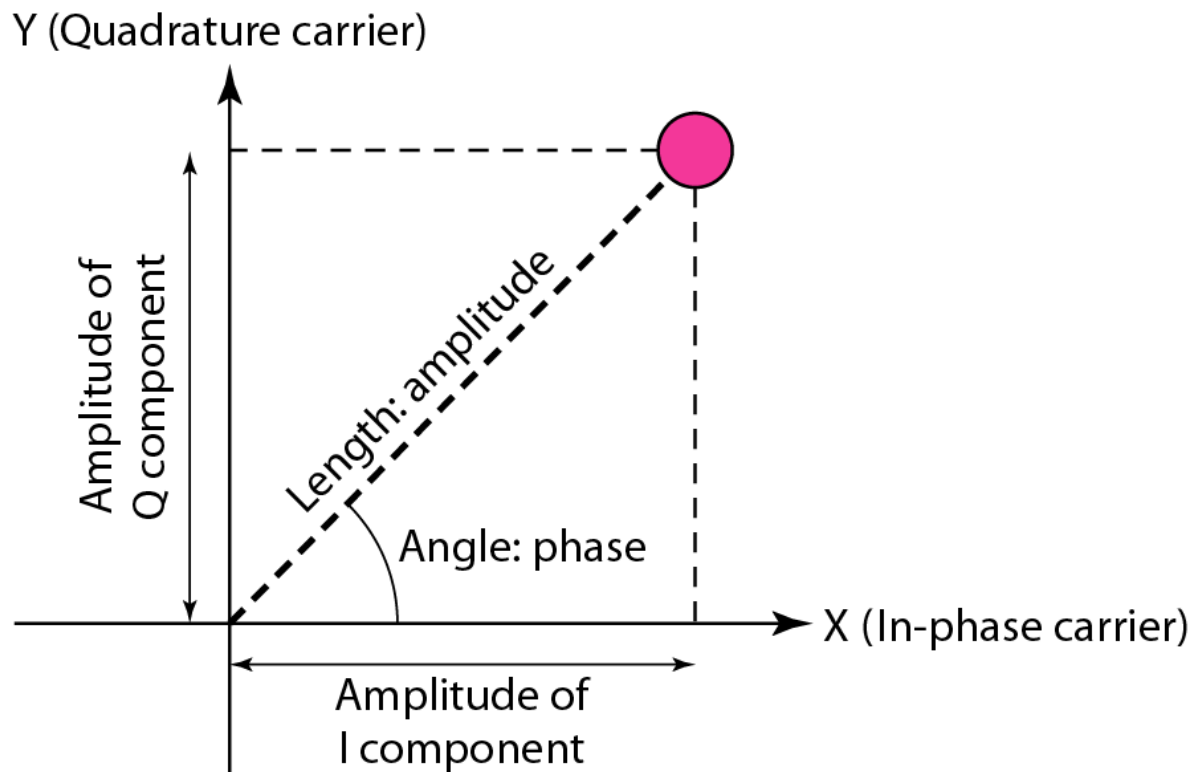


Figure 5.7 Modulation of Analog Signals for Digital Data

Constellation Diagrams

- A constellation diagram helps us to define the amplitude and phase of a signal when we are using two carriers, one in quadrature of the other.



The X-axis represents the in-phase carrier and the Y-axis represents quadrature carrier.

- For more efficient use of bandwidth Quadrature Phase Shift Keying (QPSK)

$s(t)$

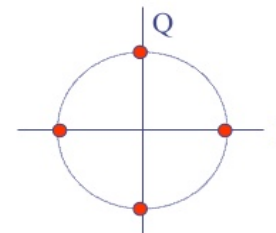
$$= A \cos(2\pi f_c t) \quad \text{for } 00$$

$$= A \cos(2\pi f_c t + 90) \quad \text{for } 01$$

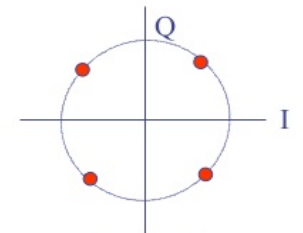
$$= A \cos(2\pi f_c t + 180) \quad \text{for } 10$$

$$= A \cos(2\pi f_c t + 270) \quad \text{for } 11$$

QPSK Constellation Diagram



Carrier phases
 $\{0, \pi/2, \pi, 3\pi/2\}$

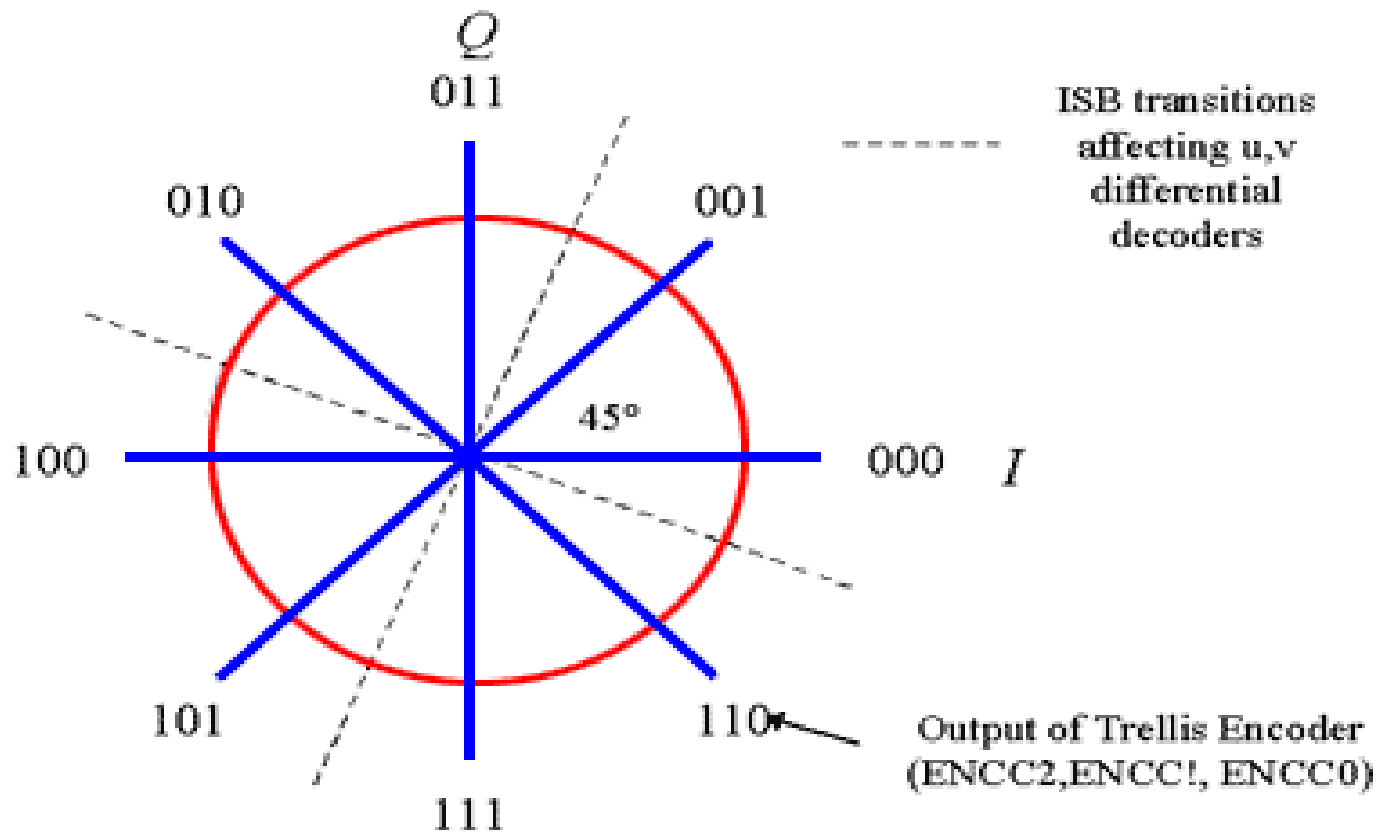


Carrier phases
 $\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}$

- ◆ Quadrature Phase Shift Keying has **twice the bandwidth efficiency of BPSK** since 2 bits are transmitted in a single modulation symbol

8-PSK

- The idea can be extended to have 8-PSK.
- The phase is shift by 45° .

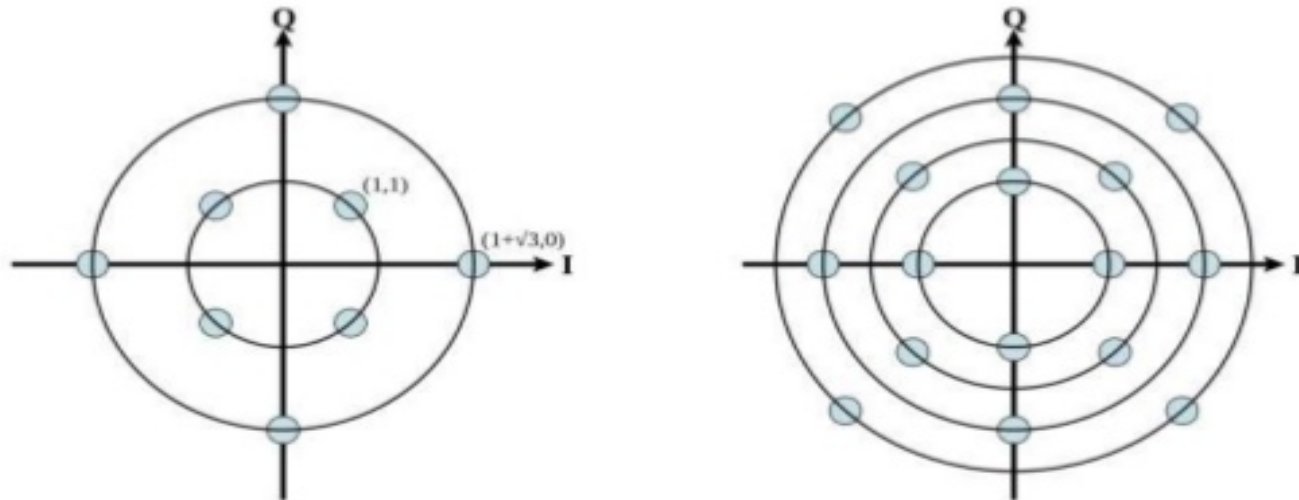


Quadrature Amplitude Modulation



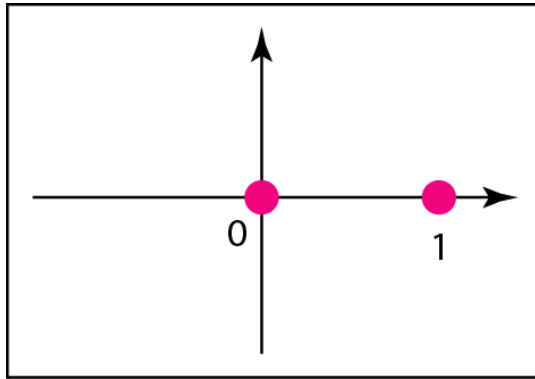
- Ability of equipment to distinguish small difference in phase limits the potential bit rate
- By combining **ASK** and **PSK** it is possible to obtain higher data rate.
- This combine technique is known as **QAM** (Quadrature Amplitude Modulation)

QAM Modulation

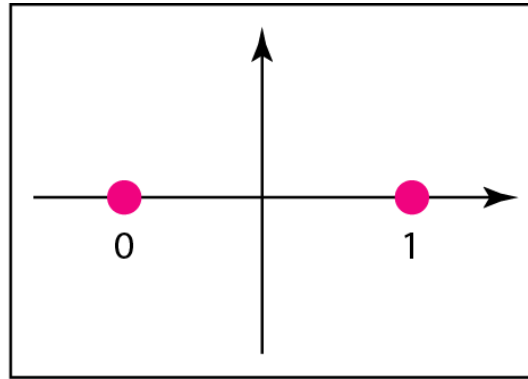


- Peek some fixed set of complex amplitude points and encode your information by switching carrier between these points.
- Such set is called **QAM constellation**.
- Each point encodes several bits and called **QAM symbol**.
- The more points are packed in QAM symbol the faster the information will be transferred, but symbols with many points are sensible to noise. So, balance is needed.

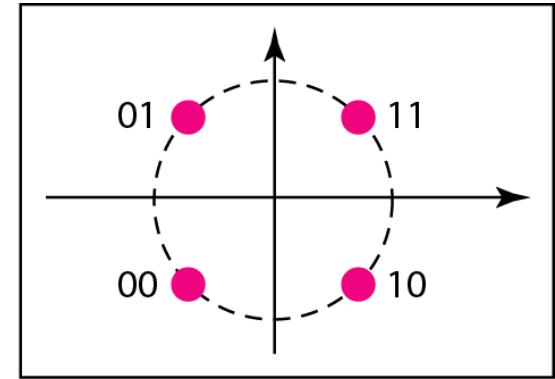
Constellation Diagrams



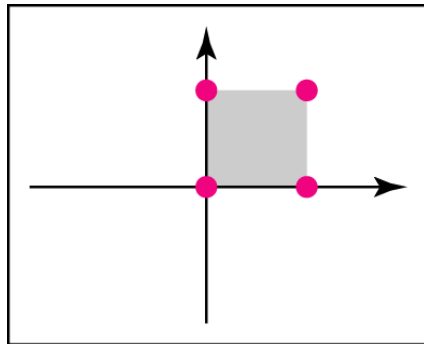
a. ASK (OOK)



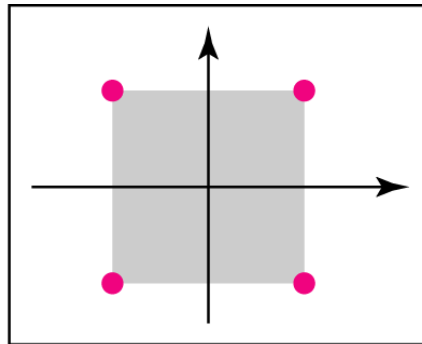
b. BPSK



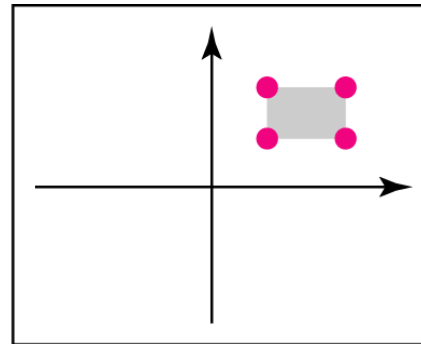
c. QPSK



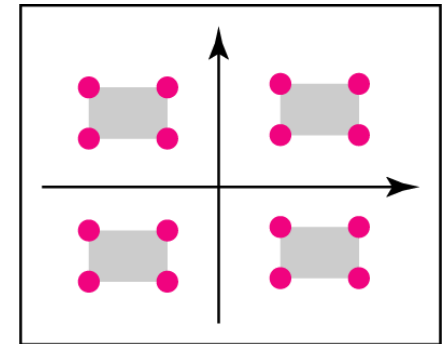
a. 4-QAM



b. 4-QAM



c. 4-QAM



d. 16-QAM

Modem



- **Modem: Modulator + Demodulator**
- Converts **digital signal into analog signal** using ASK, FSK, PSK and QAM.
- Important parameters
 - Transmission rate
 - Bandwidth (Baud rate)

Thanks!

Figure and slide materials are taken from the following sources:

1. W. Stallings, (2010), [Data and Computer Communications](#)
2. [NPTL lecture](#) on Data Communication, by Prof. A. K. Pal, IIT Kharagpur
3. B. A. Forouzan, (2013), [Data Communication and Networking](#)