CS311: Data Communication

Spread Spectrum

by

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Spread Spectrum

- essential idea is to spread the information signal over a wider bandwidth
- an important form of encoding for wireless communications
- can be used to transmit either analog or digital data, using an analog signal
- **Types**:
  - frequency hopping spread spectrum (**FHSS**)
  - direct sequence spread spectrum (**DSSS**)

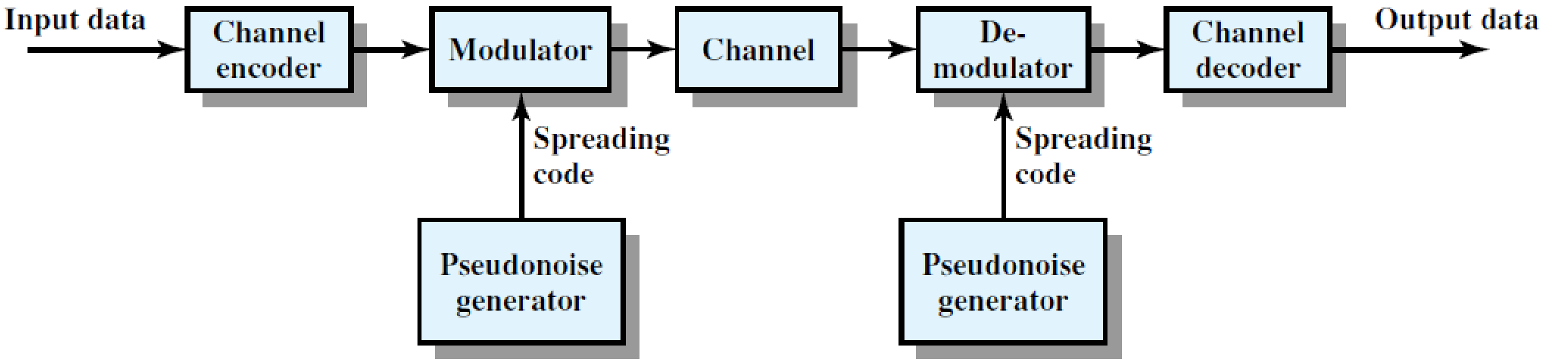


Figure 9.1 General Model of Spread Spectrum Digital Communication System

Spread Spectrum

- Pseudorandom numbers
  - generated by an algorithm using some initial value called the seed
  - produce sequences of numbers that are not statistically random, but passes reasonable tests of randomness
  - unless you know the algorithm and the seed, it is impractical to predict the sequence

- Advantages over apparent waste of spectrum
  - The signals gains immunity from various kinds of noise and multipath distortion.
  - Immune to jamming attack
  - It can also be used for hiding and encrypting signals.
  - Several users can independently use the same higher bandwidth with very little interference. (e.g. CDMA)

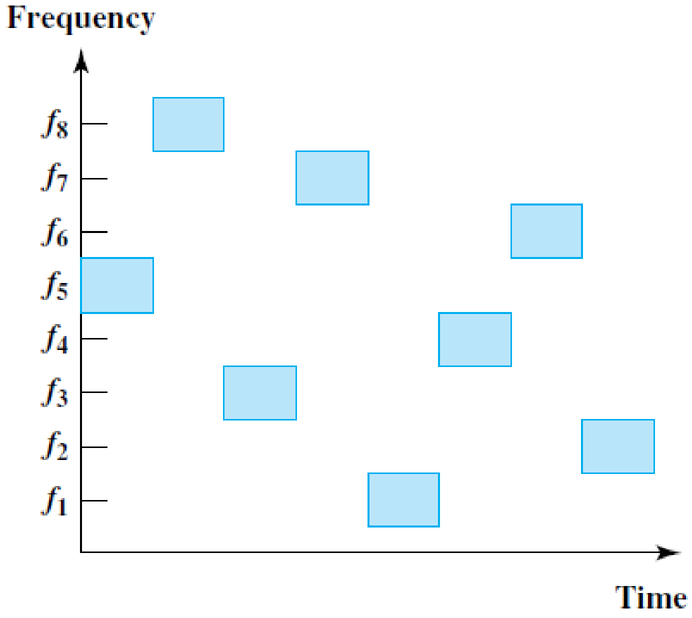
FHSS

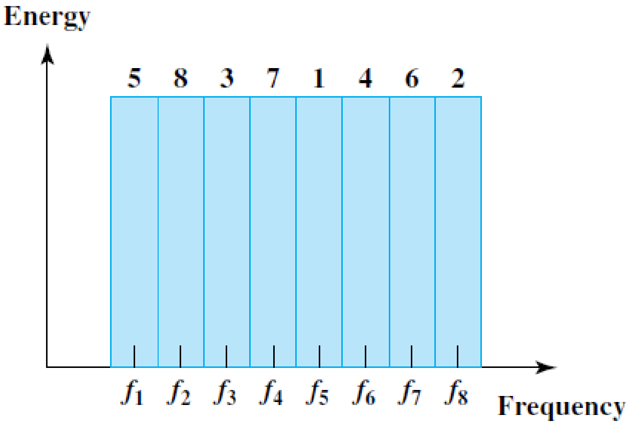
- the signal is broadcast over a seemingly random series of radio frequencies, hopping from frequency to frequency at fixed intervals.

- The transmitter operates in one channel at a time for a fixed interval

- A receiver, hopping between frequencies in synchronization with the transmitter, picks up the message

- width of each channel usually corresponds to the bandwidth of the input signal





(a) Channel assignment

BFSK

BFSK modulated signal:

$$s(t) = \begin{cases} A\cos(2\pi f_1 t) & \text{binary 1} \\ A\cos(2\pi f_2 t) & \text{binary 0} \end{cases}$$

where f_1 and f_2 are typically offset from the carrier frequency f_c by equal but opposite amounts

BFSK modulated signal:

$$s_d(t) = A\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f)t) \qquad \text{for } iT < t < (i+1)T$$

where

A = amplitude of signal

f_0 = base frequency

b_i = value of the ith bit of data (+1 for binary 1, −1 for binary 0)

Δf = frequency separation

T = bit duration; data rate = $1/T$

Thus, during the i-th bit interval, the frequency of the data signal is:

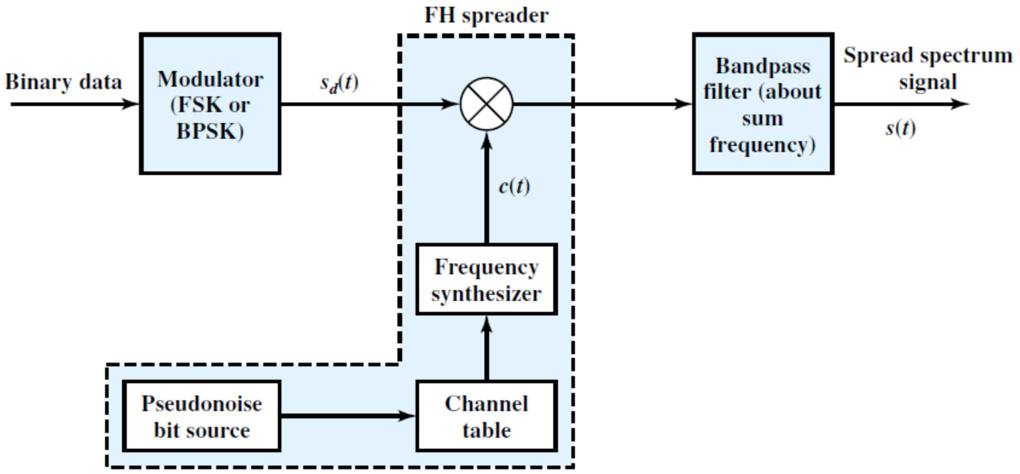
f_0 if the data bit value is -1

$f_0+\Delta f$ if the data bit value is +1

FHSS

FSK + Spread Spectrum

The frequency synthesizer generates a constant-frequency tone whose frequency hops among a set of 2^k frequencies, with the hopping pattern determined by *k* bits from the PN sequence.



(a) Transmitter

product of data signal and spreading signal during the *i*-th hop (during the *i*-th bit) is

$$\begin{aligned} p(t) &= s_d(t)c(t) = A\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f)t)\cos(2\pi f_i t) \\ &= 0.5A\,[\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f + f_i)t) \\ &\qquad + \cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f - f_i)t)] \end{aligned}$$

where f_i is the frequency of the signal generated by the frequency synthesizer during the *i*-th hop

A bandpass filter blocks the difference frequency and pass the sum frequency, we get the FHSS signal:

$$s(t) = 0.5A\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f + f_i)t)$$

Cont…

Thus, during the *i*-th bit interval, the frequency of the data signal is

$f_0 + f_i$ if the data bit value is -1

$f_0 + f_i + \Delta f$ if the data bit value is +1

- **At the receiver**: multiplied by a replica of the spreading signal

$$
\begin{aligned}
s(t)c(t) &= 0.5A\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f + f_i)t)\cos(2\pi f_i t) \\
&= 0.25A[\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f + f_i + f_i)t) \\
&\qquad + \cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f)t)]
\end{aligned}
$$

A bandpass filter blocks the sum frequency and pass the difference frequency, we get the data signal:

$$s_d(t) = 0.25A\cos(2\pi(f_0 + 0.5(b_i + 1)\,\Delta f)t)$$

FHSS Using MFSK

- Multiple FSK uses $M=2^L$ different frequencies to encode the digital input L bits at a time

- Transmitted Signal $s_i(t) = A \cos 2\pi f_i t, \quad 1 \le i \le M$

$$f_i = f_c + (2i - 1 - M)f_d$$

f_c = denotes the carrier frequency

f_d = denotes the difference frequency

M = number of different signal elements $= 2^L$

L = number of bits per signal element

- For FHSS, the MFSK signal is translated to a new frequency every T_c seconds by modulating the MFSK signal with the FHSS carrier signal.

- For a data rate of R, the duration of a bit is $T=1/R$ seconds and the duration of a signal element is $T_s = LT$ seconds.

Slow-frequency-hop spread spectrum	$T_c \geq T_s$
Fast-frequency-hop spread spectrum	$T_c < T_s$

$T_c > T_s$

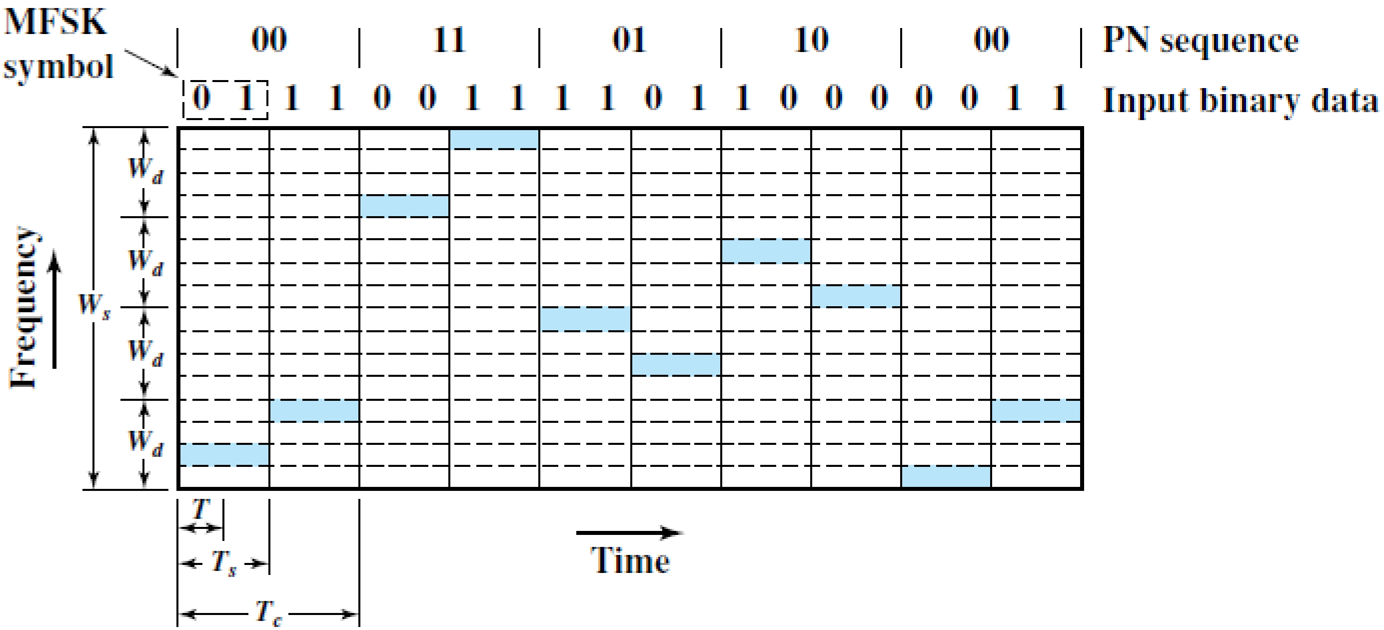


Figure 9.4 Slow Frequency Hop Spread Spectrum Using MFSK ($M = 4, k = 2$)

- we have M=4 which means that four different frequencies are used to encode the data input 2 bits at a time
- Each signal element is a discrete frequency tone, and the total MFSK bandwidth is $W_d = M f_d$
- We use an FHSS scheme with k=2. That is, there are $2^k = 4$ different channels, each of width W_d
- The total FHSS bandwidth is $W_s = 2^k W_d$
- $T_c = 2T_s = 4T$

Tc < Ts

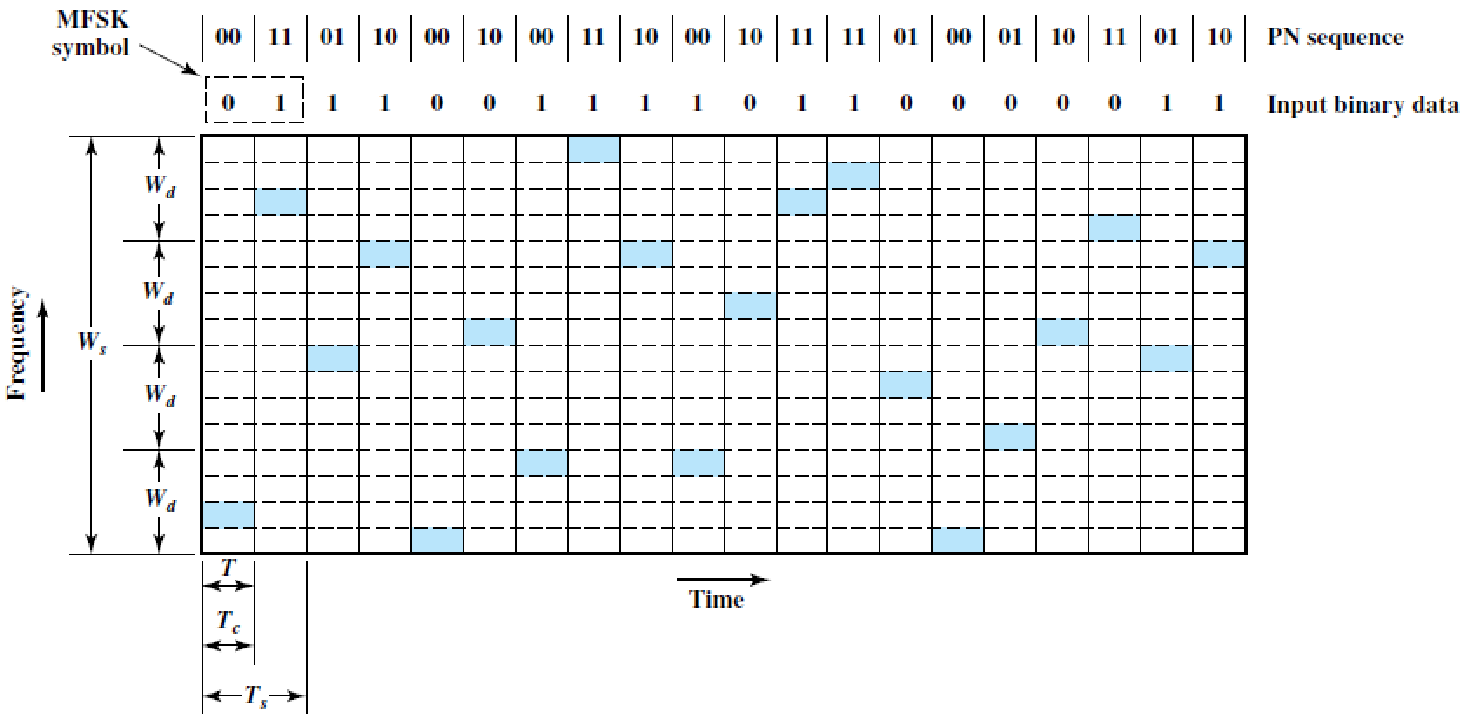


Figure 9.5 Fast Frequency Hop Spread Spectrum Using MFSK ($M = 4, k = 2$)

- we have M=4 which means that four different frequencies are used to encode the data input
- each signal element is represented by two frequency tones
- Each signal element is a discrete frequency tone, and the total MFSK bandwidth is $W_d = M f_d$
- We use an FHSS scheme with k=2. That is, there are $2^k = 4$ different channels, each of width W_d
- The total FHSS bandwidth is $W_s = 2^k W_d$
- $T_s = 2T_c = 4T$

FHSS Performance Considerations

- a large number of frequencies is used in FHSS
- So, W_s is much larger than W_d

- suppose we have an MFSK transmitter with bandwidth W_d
- a jammer of the same bandwidth and fixed power S_j on the signal carrier frequency

- we have a ratio of signal energy per bit to noise power density per Hertz of

$$\frac{E_b}{N_j} = \frac{E_b W_d}{S_j}$$

- If frequency hopping is used, the jammer must jam all 2^k frequencies.

- With a fixed power, this reduces the jamming power in any one frequency band to $S^j/2^k$

- The gain in signal-to-noise ratio, or processing gain, is

$$G_P = 2^k = \frac{W_s}{W_d}$$

DSSS

- direct sequence spread spectrum (DSSS),
  - each bit in the original signal is represented by multiple bits in the transmitted signal, using a spreading code
  - spreading code spreads the signal across a wider frequency band in direct proportion to the number of bits used

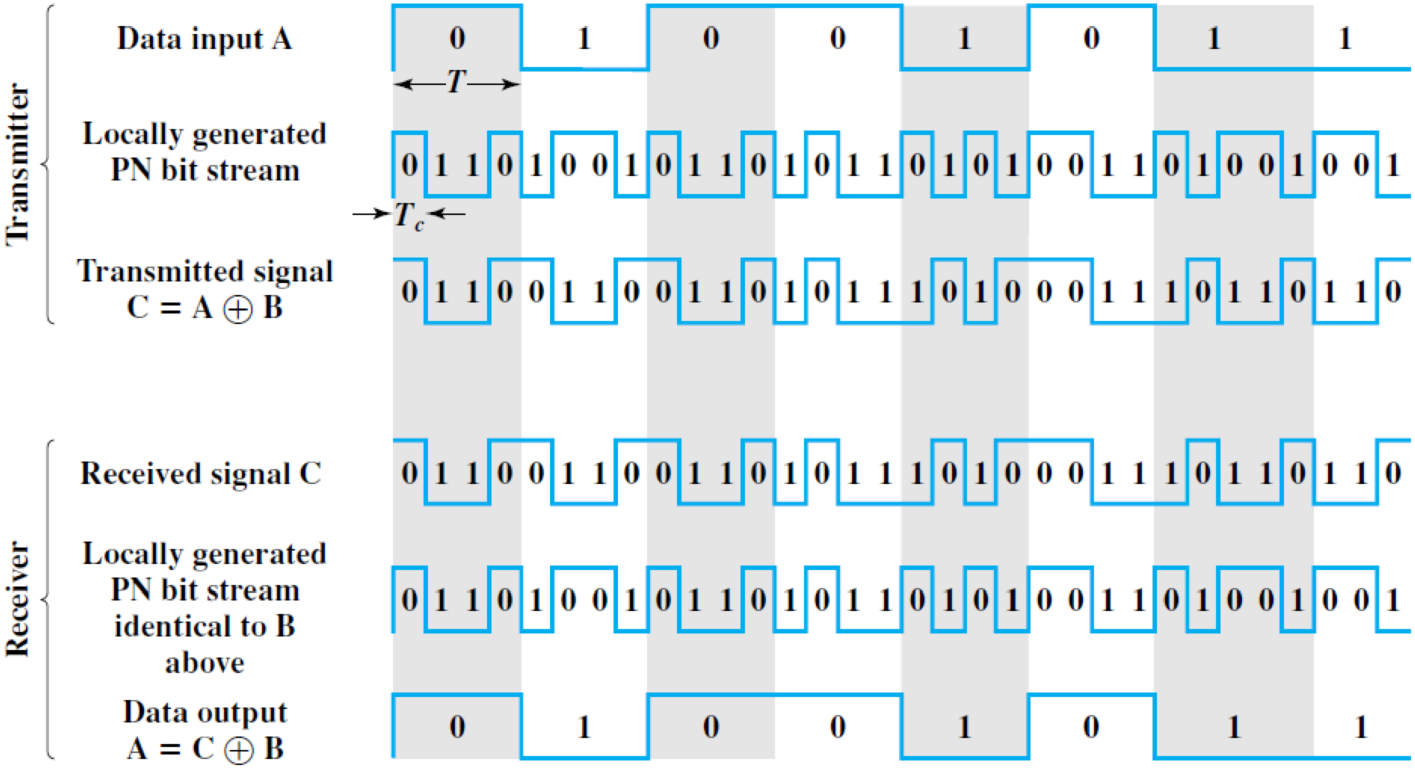


Figure 9.6 Example of Direct Sequence Spread Spectrum

DSSS System

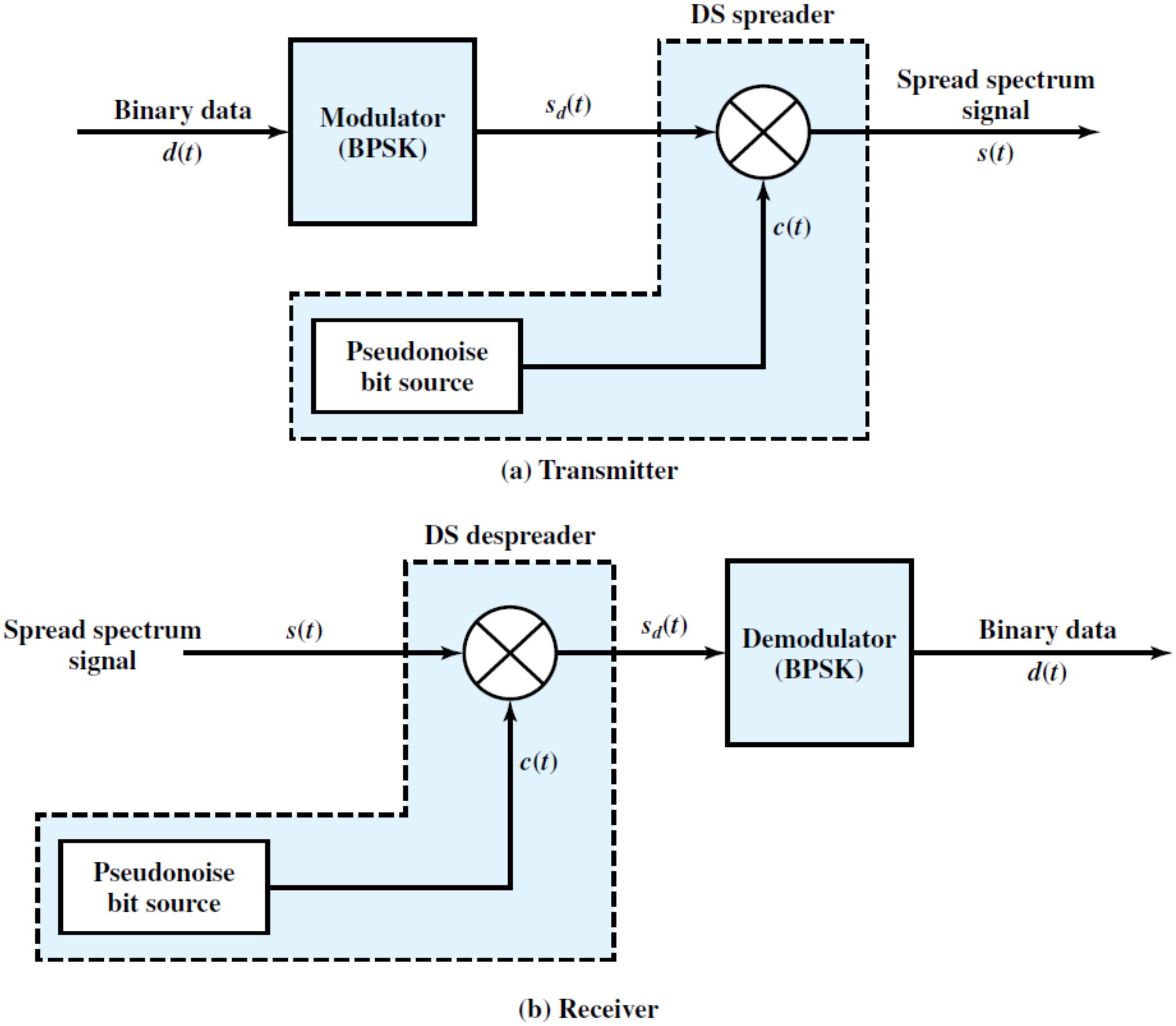


Figure 9.7 Direct Sequence Spread Spectrum System

DSSS using BFSK

- Let a BPSK signal $s_d(t) = Ad(t)\cos(2\pi f_c t)$

where,

A= amplitude of signal,

f_c = carrier frequency,

$d(t)$ = the discrete function

$d(t)$ = +1 if the corresponding bit in the bit stream is 1

$d(t)$ = -1 if the corresponding bit in the bit stream is 0

- the DSSS signal $s(t) = A\,d(t)c(t)\cos(2\pi f_c t)$

where, c(t) is the PN sequence taking on values +1 and -1.

- At the receiver, the incoming signal is multiplied again by *c(t)*.
- *c(t)* x *c(t)* =1 and therefore the original signal is recovered.

Cont…

Two interpretation at the receiver:

- first multiply *d(t)* and *c(t)* together and then perform the BPSK modulation
- first perform the BPSK modulation on the data stream *d(t)* to generate the data signal $s_d(t)$. This signal can then be multiplied by *c(t)*.

Second one

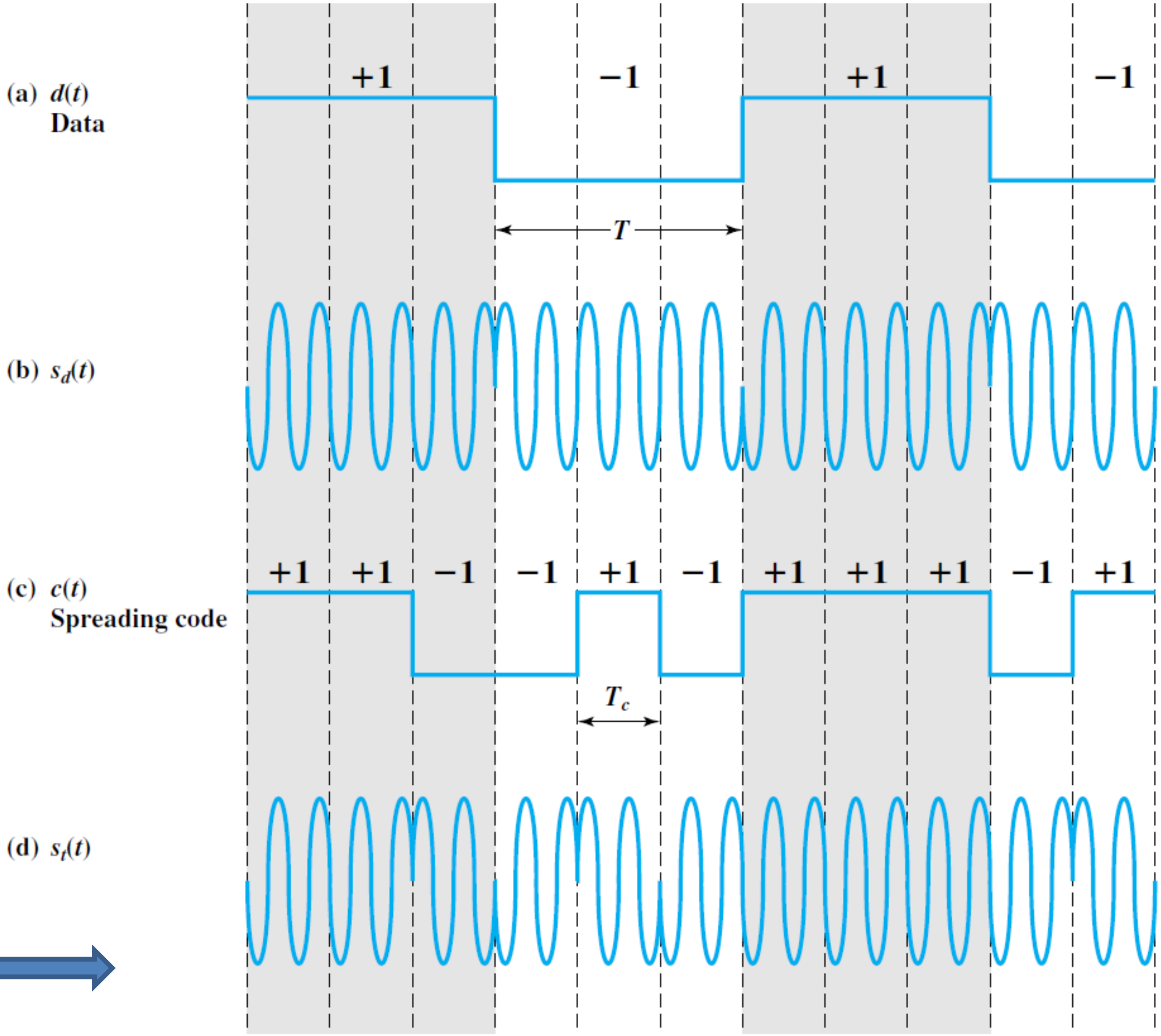


Figure 9.8 Example of Direct-Sequence Spread Spectrum Using BPSK

DSSS Performance Considerations

- Let us assume a simple jamming signal at the center frequency of the DSSS system.
- The jamming signal has the form $s_j(t) = \sqrt{2S_j}\cos(2\pi f_c t)$

  and the received signal is $s_r(t) = s(t) + s_j(t) + n(t)$

  where,

$$\begin{aligned} s(t) &= \text{transmitted signal} \\ s_j(t) &= \text{jamming signal} \\ n(t) &= \text{additive white noise} \\ S_j &= \text{jammer signal power} \end{aligned}$$

- The despreader at the receiver multiplies $s_r(t)$ by $c(t)$.
- so the signal component due to the jamming signal is $y_j(t) = \sqrt{2S_j}c(t)\cos(2\pi f_c t)$
- Thus, the jamming carrier power S_j is spread over a bandwidth of approximately $2/T_c$.

Cont…

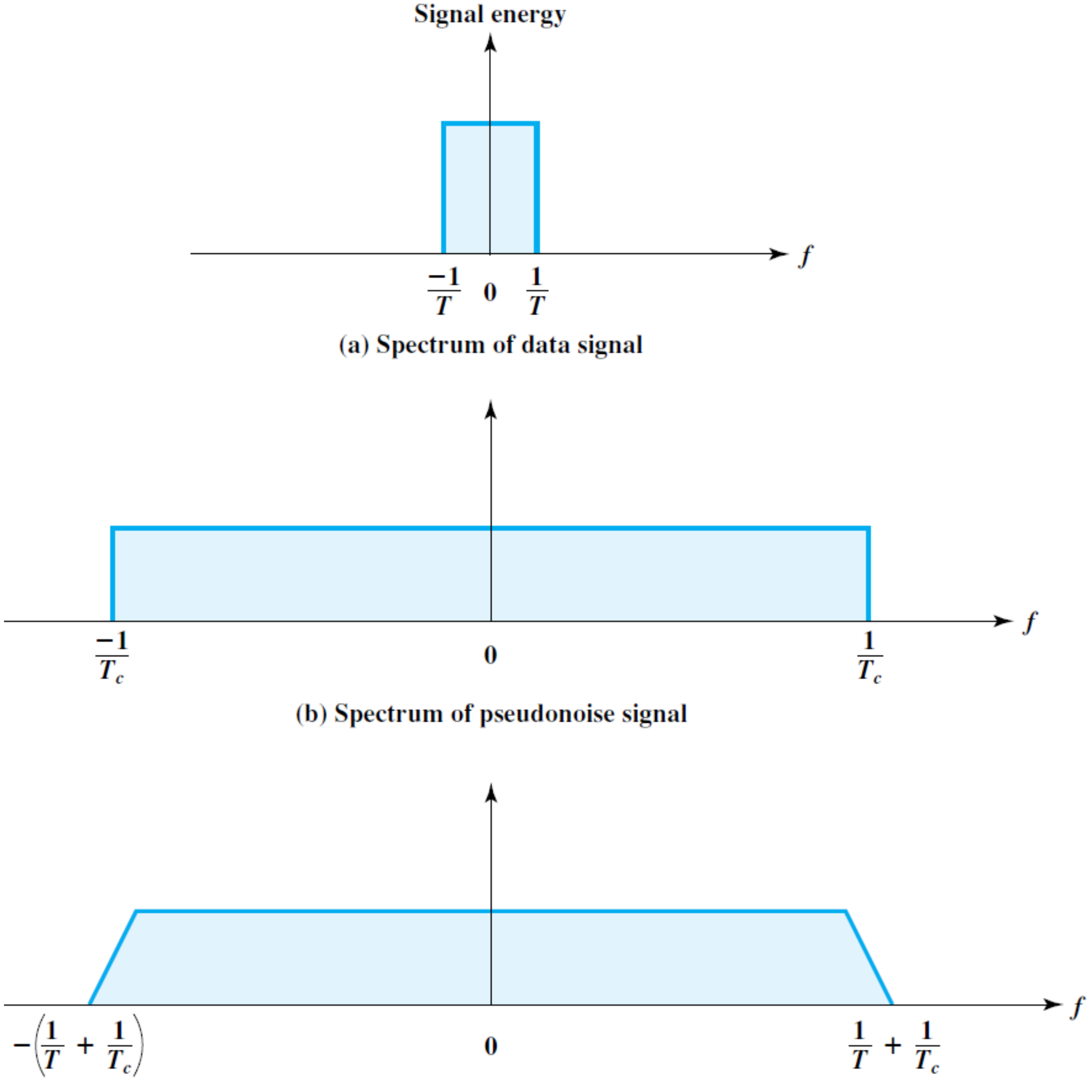
- the BPSK demodulator includes a bandpass filter matched to the BPSK data, with bandwidth of 2/T
- Thus, most of the jamming power is filtered out.
- the jamming power passed by the filter is

$$S_{jF} = S_j(2/T)/(2/T_c) = S_j(T_c/T)$$

- The jamming power has been reduced by a factor of (T_c/T)
- the gain in signal-to-noise ratio

$$G_P = \frac{T}{T_c} = \frac{R_c}{R} \approx \frac{W_s}{W_d}$$

where, R_c is the spreading bit rate, R is the data rate, W_d is the signal bandwidth, and W_s is the spread spectrum signal bandwidth.



(a) Spectrum of data signal

(b) Spectrum of pseudonoise signal

(c) Spectrum of combined signal

Thanks!

Figure and slide materials are taken from the following sources:

1. W. Stallings, (2010), Data and Computer Communications
2. NPTL lecture on Data Communication, by Prof. A. K. Pal, IIT Kharagpur
3. B. A. Forouzan, (2013), Data Communication and Networking